A simple model of gas production from hydrofractured horizontal wells in shales

Tad Patzek, Frank Male, and Michael Marder

ABSTRACT
Assessing the production potential of shale gas can be assisted by constructing a simple, physics-based model for the productivity of individual wells. We adopt the simplest plausible physical model: one-dimensional pressure diffusion from a cuboid region with the effective area of hydrofractures as base and the length of horizontal well as height. We formulate a nonlinear initial boundary value problem for transient flow of real gas that may sorb on the rock and solve it numerically. In principle, solutions of this problem depend on several parameters, but in practice within a given gas field, all but two can be fixed at typical values, providing a nearly universal curve for which only the appropriate scales of time in production and cumulative production need to be determined for each well. The scaling curve has the property that production rate declines as one over the square root of time until the well starts to be pressure depleted, and later it declines exponentially. We show that this simple model provides a surprisingly accurate description of gas extraction from 8305 horizontal wells in the United States’ oldest shale play, the Barnett Shale. Good agreement exists with the scaling theory for 2133 horizontal wells in which production started to decline exponentially in less than 10 yr. We provide upper and lower bounds on the time in production and original gas in place.
ripples at the sea bottom. He specializes in the mechanics of solids, particularly the fracture of brittle materials. He has developed numerical methods allowing fracture computations on the atomic scale to be compared directly with laboratory experiments on a macroscopic scale. He is employing these methods to study the production of natural gas from hydrofractured shale. He has also published two textbooks, one a graduate text on condensed matter physics, and the other an undergraduate text on research methods for science.

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NOMENCLATURE
Symbols and dimensions of key quantities
Symbol | SI dimensions | Field dimensions
---|---|---
c | Pa$^{-1}$ | μ cip
$d$ | m | ft

$D$-production decline coefficient
$k$-permeability
$m$-gas pseudopressure
$\nu$-cumulative produced mass
$M$-Original gas in place
$M$-molecular weight
$H$-formation thickness
$p$-pressure
$q$-volumetric flow rate
$Q$-volumetric cumulative production
$R$-universal gas constant
$RF$-recovery factor
$S$-saturation
t-time in production
$T$-temperature
$\nu$-specific volume
$y$-molar composition
$Z$-compressibility factor
$\alpha$-hydraulic diffusivity
$\kappa$-dimensionless constant for gas production in square root phase
$\kappa$-dimensional constant for gas production in square root phase
$\mu$-gas viscosity
$\rho$-density
$\tau$-time to interference
$\phi$-porosity

Subscripts and Superscripts
Symbol | Meaning
---|---
a | adsorbed
$f$ | (hydro)fracture
g | gas
$i$ | initial
$L$ | Langmuir
$ST$ | stock tank conditions
$w$ | water
$wc$ | connate water
$\sim$ | dimensionless
$\wedge$ | specific
$0$ | reference or standard conditions
INTRODUCTION

The fast progress of hydraulic fracturing technology has led to the extraction of natural gas and oil from tens of thousands of wells drilled into mudrock formations almost exclusively in the United States. However, a significant potential exists for drilling into mudrock formations on all continents (Birol, 2012). The fracturing technology has generated considerable concern about environmental consequences (Osborn et al., 2011; Vidic et al., 2013), and about whether hydrocarbon extraction from mudrocks will ultimately be profitable (Hughes, 2013).

The cumulative gas obtained from the wells hydrofractured in mudrocks and their ultimate profitability depend upon how the hydrocarbon flow rate varies over time. Large-scale use of hydraulic fracturing in mudrocks is relatively new, and therefore data on the behavior of hydrofractured wells on the scale of 10 yr or more are only now becoming available.

Our aim is to assemble the basic equations governing flow of high-pressure gas in a porous medium and assess whether gas transport in mudrock beds can adequately be described by a diffusion equation written and solved in one dimension. We believe this to be the most appropriate setting, although the most familiar reservoir flow laws result from flow problems posed in two dimensions (Kelkar, 2008). We address this question by comparing simple model flow problems with data from a large subset of all 16,533 vertical and horizontal wells in the Texas’ Barnett Shale. The Barnett Shale is a Mississippian mudstone located in the Bend arch–Fort Worth Basin.

The power-law behavior of the flow is most compatible with a one-dimensional (1-D) model. However, we will show that the rate at which the flow occurs is incompatible with laboratory values of gas diffusivity in shale. Therefore, the subsurface geometry created by hydrofracturing and reactivation of microcracks, natural fractures, and faults is complex, as observed by Olson (2004), Gale et al. (2007), and Geiser et al. (2012), among others. Our understanding of the nature, creation, and evolution of hydrofractured natural fracture systems is far from complete.

Recently, we presented the derivation of a simple model that appears to capture the essential features of gas production from tight shale formations (Patzek et al., 2013). Here, we expand on the conclusions of that work, particularly by exploring the experimental foundations more thoroughly. The structure of this paper is as follows. After a brief discussion of the mostly heuristic models of decline of well production, we present the conceptual structure of our physics-based model of gas production. The model constitutes the simplest description consistent with the basic physics of gas extraction from horizontal fractured wells. The basic idea of the model had previously been presented by Al-Ahmadi et al. (2010). Nobakht et al. (2012) found analytical approximations they compared with numerical solutions and a small number of field cases. In Patzek et al. (2013), we provided a detailed analysis showing that the exact solution of the recovery factor in this model reduces to a nearly universal scaling curve that can be tabulated for convenient comparison with field data. Here, we discuss properties of the dimensionless recovery factor and fit it to a mean horizontal well in the Barnett, yielding effective rock permeability and hydrofracture area. Next we analyze a group of 66 good (producing most of the time, not converted, and not refractured) horizontal wells in the Barnett Shale that evidently were in transient flow for up to 8 yr, and we describe a composite median well that we use to test the validity of our assumptions. We also analyze a group of 105 similarly defined good vertical wells, and show that most wells in the Barnett Shale have been in 1-D (rectilinear) transient flow for up to 7 to 10 yr. Finally, we apply our model to 8305 horizontal wells that have not been recompleted and generate distributions of their times to exponential decline of production rate caused by pressure interference, original gas in place, hydrofracture area, and drainage area.

Brief History of Predictions of Well Production

Attempts to quantify the rate of decline of oil and natural gas production by a well are as old as reservoir engineering. In 1944, J. J. Arps first published a seminal paper on several types of decline of well production: exponential, hyperbolic, harmonic, and geometric (Arps, 1945). In that paper, Arps reviewed almost 30 yr of the prior history of empirical
predictions of future well production. Thirty-six years later, Fetkovich (1980) showed how the different well flow equations listed by Arps arise from radial transient flow.

Today, the literature discussing production from oil, gas and water wells is enormous, and impossible to summarize briefly. A Google search with “frac*” conducted on November 4, 2013, returned over 143 million hits. Another Google Scholar search for any of the keywords, gas, well, shale, and/or production, returned 156,000 research papers. Nevertheless, the long and continuing tradition of using purely phenomenological models in this area means that putting effort into models based upon physical reasoning is worthwhile.

Jones (1942), also quoted by Arps, suggested for wells declining at variable rates an approximation whereby the fractional decline-time relationship is given by a straight line in log–log coordinates:

\[ \ln D = \ln D_0 - m \ln t \]  
(1)

in which \( D \) is the fractional decline coefficient of fluid production rate, \( D_0 \) is the initial value of that coefficient, \( t \) is time in production, and \( m \) is the slope; see Section 1 for nomenclature.

If \( a \) is an increment of the logarithm of volumetric production rate \( q \) over the \( n^{th} \) production time increment \( \Delta t \):

\[ a_n = -\left( \frac{1}{q} \frac{dq}{dt} \Delta t \right)_n > 0, \quad n = 0, 1, 2, \ldots \]  
(2)

then the production decline coefficient \( D = \frac{1}{a} \) is a constant for exponential decline; \( D \) is proportional to \( q^b \), \( 0 < b < 1 \), for hyperbolic decline; proportional to \( q^1 \) in harmonic decline; and \( D \) diminishes with time at a constant fraction in geometric decline,

\[ D_n = \frac{r^{-\sum \delta t_i}}{a_n/a_{n-1}} \]  
(3)

in which \( r \) is the constant decline ratio, \( n = 0, 1, 2, \ldots \) is the counter of the subsequent production time (with well downtimes excluded) intervals \( \delta t_i \), months in the current paper.

Upon integration, equation 1 gives:

\[ q = q_0 \exp \left( -\frac{D_0 t^{1-m}}{1-m} \right), \quad 0 \leq m < 1 \]  
(4)

in which \( q_0 \) is the initial production rate. If we set \( n = 1 - m \) and define a characteristic interference time, \( \tau \),

\[ \tau = \left( \frac{D_0}{n} \right)^{-1/n} \]  
(5)

we recover the stretched exponential equation of Valko and Lee (2010):

\[ q = q_0 \exp \left[ -\left( \frac{t}{\tau} \right)^n \right] \]  
(6)

proposed to explain the expected rates of production decline for large groups of wells in the Barnett Shale.

Arps (1945, p. 229) already stated that the various estimates of production decline are based on the assumption that future well behavior will be governed by whatever trend or mathematical relationship is apparent in its past performance:

“...This assumption puts the extrapolation methods on a strictly empirical basis and it must be realized that this may make the results sometimes inferior to the more exact volumetric methods.”

The volumetric methods mentioned by Arps are based on geology, core analysis, wireline logging, and downhole well pressures.

Semi-analytic and numerical models of transient flow in vertical wells with vertical hydrofractures have been developed for 50 yr under a variety of assumptions. Most of the relevant papers have been devoted to short- and intermediate-time well testing: pressure build up, well storage effects, and flow tests. The classical papers by Prats (1961), Russell and Truitt (1964), Wattenbarger and Ramey (1969), Gringarten and Henry (1974) should be mentioned in this context. A 1-D (rectilinear) transient flow analysis was also applied to long-time production from hydrofractured wells in ultra-low-permeability formations. Patzek (1992) analyzed performance of over 1000 production and injection wells in the one microdarcy, naturally fractured South Belridge Diatomite oil field in California, and demonstrated the significant extensions of injection hydrofractures and injector–producer linkages. Most wells remained in purely transient flow for more than a decade. Wattenbarger et al. (1998) have calculated decline curves for tight gas wells and concluded that
rectilinear flow may last for 10 or 20 yr. Kelkar (2008) in his Chapter 4 discusses 1-D (rectilinear) and two-dimensional (2-D) (radial) flow models, with an example drawn from the Barnett Shale. Similar to many other authors, Kelkar focuses on flow from a semi-infinite region into a plane, avoiding interference effects that will be prominent in our analysis.

Nobakht and Clarkson (2011a) looked at tight gas wells and gas shale wells in 1-D (rectilinear) transient flow and used production data to calculate the effective formation permeability and fracture half-lengths. By effective permeability, we mean the enhanced permeability of the shales produced by the hydrofracturing process, which in the current paper we will treat as spatially uniform. Nobakht and Clarkson (2011a) have concluded that not accounting for gas slippage in nanopores may lead to an underestimated rock permeability and overestimate of size of the hydrofractures. However, their own calculations show a negligible impact of slip, well within the noise in the field data. In addition, Nobakht and Clarkson (2011b) accounted for variable fluid and rock properties. They used numerical examples in which neither the flow rate nor flowing wellbore pressure were constant. The problem with this approach is that reservoir pressure is not well known in shales, and well flowing pressures are not reported accurately or not at all over multi-year time intervals (Lee and Sidle, 2010).


The physical model underlying time-dependent 1-D flow was presented conceptually by Al-Ahmadi et al. (2010) and solved in approximate analytical as well as in numerical fashion by Nobakht et al. (2012). This model, we maintain, is the simplest possible starting point for analysis of hydrofractured wells that is consistent with the physics of the extraction process. Nevertheless, it has received relatively little attention, and appears to be regarded as advanced.

Consider a single horizontal well, shown in Figure 1. Suppose that this well has $N$ vertical transverse hydrofractures that are spaced uniformly along its entire horizontal length. The exterior sides of the first and last hydrofracture drain the reservoir volume external to the horizontal-well length, and this additional production will also be accounted for. Essentially identical well schematics appear as figure 2 in Al-Ahmadi et al. (2010) and figure 1 in Nobakht et al. (2012).

If each hydrofracture plane in the well is separated by distance $2d$ from another fracture plane, each two consecutive fractures will interfere with one another after a certain characteristic time, (Patzek et al., 2003). For pressure-dependent fluid and formation properties, this characteristic time changes with the pressure.

**MODEL OF WELL PRODUCTION**

Until relatively recently, mudrock formations were thought unsuitable for economic gas production because the source rock has very low permeability, on the order of one nanodarcy (Vermylen, 2011). The hydrofracturing process raises the effective permeability, as we will show in this paper, by one or two orders of magnitude. However, only those spatial regions in which fractures have been activated and propped open are reasonable candidates for the enhanced permeability. Therefore, a physically reasonable starting point for the study of hydrofracture wells should consider pressure diffusion from a finite region, outside which diffusion vanishes, rather than from an infinite or semi-infinite region, as for conventional wells (Kelkar, 2008; Dake, 1978).

Thus, we pursue three goals. The first is to present a brief and largely conceptual review of the model we derived in Patzek et al. (2013). The second is to show how the solution can be used to obtain upper and lower bounds on gas production rates and recoverable reserves. The third is to perform a comprehensive analysis of the oldest shale play in the United States, the Barnett Shale, and show that the model in fact describes decline curves accurately. An accurate but practically usable model of gas production from shales is important because shale gas has pumped over $800 billion into the United States economy in the last 5 yr, whereas at the same time serious questions exist about the profitability and economic viability of extraction (Patzek et al., 2013). Both decisions about how to invest in and regulate shale gas extraction need to be informed by accurate scientific assessment.
As shown in equation 28, the reference time to pressure interference \( \tau \), or the interference time is

\[
\tau = \frac{d^2}{\alpha_i} \quad \alpha_i = \left. \frac{k}{\varphi S_g \mu_g c_g} \right|_{\text{initial reservoir, } T}
\] (7)

Equation 25 defines the hydraulic diffusivity \( \alpha_i \) in terms of the gas viscosity \( \mu_g \), compressibility \( c_g \), saturation \( S_g \), the reservoir permeability \( k \), and porosity \( \varphi \). The interference times are shown in Figure 2 for a typical spacing of hydrofractures and typical reservoir temperatures and pressures. Also see Table 1 for typical values.

Note that \( \tau \) in equation 7 is a constant defined at the initial average pressure \( p \) and temperature \( T \) of the reservoir, and does not depend on the instantaneous gas pressure that will vary in space and time as the reservoir is depleted.

In equation 28, we define the dimensionless time, \( \bar{t} \), as:

\[
\bar{t} = \frac{t}{\tau}
\] (8)

Let \( m \) be the cumulative gas production in units of mass, and let \( M \) be the mass of the original gas in place that is contained in the reservoir volume drained by a single horizontal well. Our final result for the solution of the model takes a deceptively simple form. We compute explicitly a dimensionless recovery factor, RF,

\[
RF(\bar{t}, y, T, p_i, p_f) = \frac{m}{M}
\] (9)
that depends only on the dimensionless time, the molar composition of gas \( y \), the reservoir temperature \( T \), the initial reservoir pressure \( p_i \), and the hydrofracture pressure \( p_f \).

**PROPERTIES OF RF(\( \tilde{t}, y, T, p_i, p_f \))**

Our model of single-phase fluid (e.g., gas) production from hydrofractured horizontal (and vertical) wells is presented and solved exactly in Appendices 1–3. Here, we discuss features of the solution, and its application to production data.

The recovery factor RF can be computed by solving an initial boundary value problem. We illustrate the solution first by scaling it with various characteristic values and comparing to a median good horizontal well in the Barnett Shale, Figure 3. This median well, defined in the next section, will produce 2.5 Bscf of gas in 30 yr. Because pressure interference among the hydrofractures and gas compressibility have been accounted for in the model, this prediction is quite realistic, barring a mechanical failure of the well.

![Figure 3. The dimensionless rate recovery factor RF for the median composite well in Figures 5–7.](image)

Additional plots in dimensionless form appear in Figures 15 and 16.

The solution to the initial value problem depends upon gas composition, the initial state of the reservoir \((p_i, T)\), and the hydrofracture pressure \(p_f\), but is independent of the details of the well geometry, the diffusivity, or the original gas in place.

For practical purposes, we will divide well production data into two phases that correspond to particular features of the recovery factor scaling curve. We will call them the square root phase and interference phase. The first phase corresponds to wells sufficiently early in their production history so that flow from a reservoir cell between two adjacent fractures has not yet begun to produce pressure depletion at the center of the cell. The second phase corresponds to wells for which the pressure depletion process has become visible.

The first phase is named for the fact that for dimensionless times \( \tilde{t} \) sufficiently less than 1, the recovery factor takes a particularly simple form:

\[
RF(\tilde{t}, y, T, p_i, p_f) = \kappa(y, T, p_i, p_f) \sqrt{\tilde{t}}, \quad \text{for } \tilde{t} \ll 1
\]

(10)
Here, $\kappa$ is a dimensionless constant. In general it depends on the gas composition and temperature, the limiting reservoir pressures $p_i$ and $p_f$ and is obtained from detailed solution of equation 29 in Appendix 1. For the conditions of the Barnett Shale, we have used $\kappa = 0.625$, and it has a value of about 0.6 for all reservoir conditions we have checked.

The second phase begins when depletion because of interference from two adjacent fractures causes measurable deviation from the square root growth of cumulative recovery. We have settled on the somewhat arbitrary criterion that interference is visible when the dimensionless time $\tilde{t}$ reaches a value of 0.64. For larger scaled times the growth in recovery factor slows, and it eventually reaches a plateau that describes the maximum recovery possible for the given problem parameters. The way this slowing down occurs depends in detail upon the thermodynamics of gas expansion, the reservoir permeability, and the initial and final pressures in the reservoir.

To first illustrate the use of equations 9 and 10 suppose one has an estimate of the original gas in place, $M$. The greatest uncertainty in $M$ is likely to be the effective area $A_f$ of the transverse hydrofractures. After transients of the first few months of production have subsided, cumulative production takes the form $K\sqrt{t}$ (for details see Appendix 1). Then

$$M\kappa \sqrt{t} \tau = K \sqrt{t}$$

$$\Rightarrow \tau = \left( \frac{M\kappa}{K} \right)^2$$

Thus, to estimate the interference time $\tau$ that sets the scale on which well production will begin to decline exponentially, measure $K$ from the first year of production, estimate $M$ from the well geometry, use the nominal value of $\kappa = 0.625$, and $\tau$ follows from equation 12. Here, $K$ is an empirical constant with dimension of gas mass divided by the square root of time. The first 1–3 months in production should be discounted.

The practical difficulty we face with gas production from the hydraulically fractured horizontal wells is greater than this example indicates. Neither the total mass of gas that ultimately will be extracted nor the time scale for interference to begin is known with
any precision. The original mass of gas in place is uncertain mainly because the effective hydrofracture length $2L$, and the number of active hydrofractures are uncertain. The time to interference is uncertain because the hydrofracturing process greatly increases the effective permeability $k$ of the rock in the vicinity of the well; laboratory values of $k$ obtained from core samples are on the order of nanodarcys (Vermylen, 2011), whereas accounting for observed well production requires effective values of $k$ approximately 100 times greater. When wells are in the square root phase, $\tau$ and $M$ can not be determined independently. When interference begins, both interference time $\tau$ and original gas in place $M$ can be determined through careful comparison of cumulative production data with our dimensionless recovery factor.

FIELD OBSERVATIONS FROM SELECTED WELLS

To commence a discussion of field data, we turn to a sample of 66 good horizontal wells. This sample comprises 30 Denton County wells, 10 Johnson County wells, 12 Tarrant County wells, and 14 Wise County
wells. Therefore, the sample consists of wells from the best or core area of the Barnett Shale shown in Figure 4 and characterized in Figure 5: Three 2003 wells, 18 2004 wells, and 45 2005 wells; thus, the sample wells have up to 8 yr in production. No refracturing or pressure interference occurred in any of the wells. The production rates from the 66 wells are shown in Figure 6 and their cumulative production in Figure 7.

Almost all Barnett wells are in the square root phase, not just the median composite well. The median and mean of well production are close, but the median curves are less noisy. We use the median well only for clarity, to fix ideas and to avoid getting distracted by excessive detail.

Suppose that the typical initial reservoir pressure in the Barnett is 3500 psi (24 MPa) and the horizontal wells have 12 hydrofractures on the average, spaced every 360 ft (110 m). By consulting Figures 2–7, one may conclude that the effective formation permeability is no higher than 0.5 μd, or 500 nanodarcys, and the median well in the core area of the Barnett play is expected to produce 1.64 ± 0.96 Bscf after about 7 yr in production. It also appears that the initial production in the sample wells is significantly lower than the square-root-of-time curve, because it is choked and/or water left after hydrofracturing fills the lower parts of all hydrofractures and impedes production until this water is produced and/or evaporated by gas. This low performance relative to an equivalent ideal gas well amounts to a production decrease of up to 50–150 million standard cubic feet of gas over the first 1–3 months in production.

Figure 8. Cumulative gas production versus time in production. Note that the x axis is nonlinear, because it is scaled in units of the square root of time in production. This well sample consists of 105 vertical wells; a few were converted and/or refractured, as indicated by the abrupt changes of slope that can be treated through superposition. In these coordinates, most wells plot as straight lines for up to 10 yr in production, indicating little or no interference among the vertically-spaced hydrofractures that are also vertical. Thus, one may safely assume that almost no interference occurs among the formation layers and individual wells. The median well is plotted as the thick black line. Notice the initial production lag.
**Well-by-Well Check of Model Quality**

The production from vertical and horizontal wells shown in Figures 8 and 9 follows 1-D transient flow. Flow superposition (breaks in the line slope) exists for several vertical wells, as well as a hint of interference after up to 10 yr in production. The horizontal wells are mainly perfect examples of the square root phase, except for a small subset showing evidence of interference. Therefore, if the time in production increases four-fold, the cumulative gas production $Q$ should increase two-fold. Note that this prediction does not depend on any model parameters, but only on the validity of the assumption of 1-D transient flow:

$$\frac{Q(24 \text{ months})}{Q(96 \text{ months})} = \frac{1}{2}$$

(13)

The results of this test are displayed in Figures 10 and 11. The bottom line is that our model represents the field quite well. The mean values of the cumulative production ratios are 0.45 and 0.43, respectively, for the sample of vertical and horizontal wells. The small but curious downward bias of the horizontal well data may be caused by several factors: The first month in production is on average 15 days. In addition, for the first 1–3 months, well production lags, is erratic, and it stabilizes only gradually. Patzek (1992) had to omit the first three months in production for the diatomite wells because they were producing the hydrofracture proppant and reservoir rock, and had to be cleaned. It appears that water left behind after hydrofracturing blocks lower parts of the hydrofractures and limits initial gas production until this water is produced and/or evaporated in situ. This is a real problem in all wells, because—as we
show in (Patzek et al., 2013)—the emergence of the square-root phase is universal, and the downward deviations from this regime stem from the problems with starting field wells properly.

Another interesting comparison is with the declines of most Barnett Shale wells grouped by Valko and Lee (2010). In their Figure 4, these authors show the ratios of cumulative gas production after 2 and 1 yr, 3 and 1 yr, respectively, for what they term month-groups of all vertical and all horizontal wells. Their ratios are stable, but not quite $\sqrt{2} = 1.41$ and $\sqrt{3} = 1.73$ as required by our model. However, if one discounts the first 3.5 months of production from all wells, by putting

$$x = \frac{3.5}{12}$$

$$R_{2/1} = \sqrt{\frac{2-x}{1-x}} = 1.55$$

$$R_{3/1} = \sqrt{\frac{3-x}{1-x}} = 1.96$$

one recovers table 2 in Valko and Lee (2010). Here, $R_{2/1}$ and $R_{3/1}$ are the ratios of cumulative gas production after 2 and 3 yr, respectively, relative to that after 1 yr. The same procedure can be applied to the wells shown in Figures 10 and 11, and the model-predicted ratio is 0.47.

FIELD OBSERVATIONS FROM COMPLETE BARNETT FIELD

We now set out to answer the following question: Can one extract enough information from existing noisy field production data to estimate both the interference time $\tau$ and the original gas in place $M$ at the same time? In the early stages of gas production, when $t \ll \tau$, the production rate declines purely as $1/\sqrt{t}$ and this cannot be done. Wells delivering a small ultimate amount of gas at a relatively high rate cannot be distinguished from those in which lower permeability rock or a small number of hydrofractures delivers ultimately larger quantities of gas at a relatively lower rate. Only the onset of interference between adjacent hydrofractures makes it possible to disentangle the two scenarios.

We obtained data for 16,533 wells in the Barnett Shale, and from them selected the 8807 horizontal wells that had operated continuously for 18 months or more, and had not been recompleted (the hydrofracturing process was not repeated to increase production). We plot all these wells in Figure 12. Note
the excellent agreement overall. As shown in Patzek et al. (2013) the wells in this plot fall into two basic groups. Interference is sufficiently advanced in 2133 wells that it can be detected with an average parameter uncertainty of less than 20%. These wells have begun to interfere. The remaining 6172 wells for which interference is not yet visible are still growing simply as the square root of time.

Figure 13 provides four additional pieces of information for all the wells. In the upper left panel we provide a lower bound on the interference time $\tau$. This lower bound is obtained by noting that interference becomes evident when $\tilde{t}$ reaches 0.64, so if interference is not evident, the interference time $\tau$ must be at least 1.6 times larger than the current life of the well. From this estimate one obtains a lower bound on the gas in place for each well, because equation 12 and the known value of $K$ for each well turns a lower bound on $\tau$ into a lower bound on $M$ (upper right). The lower right panel displays an upper

Figure 12. Comparison of 8305 wells with scaling function. Data come from the Barnett Shale, scaled so as to fit our scaling function (initial reservoir pressure of 3500 psi [24 MPa], hydrofracture pressure of 500 psi [3.4 MPa]). Burnt orange curves give scaled production of each well, and the black curve is the scaling function. Overall agreement is satisfactory.

Figure 13. Bounds on the interference time $\tau$ and the original mass of gas in place $M$ for the wells from Figure 12.
bound on the original gas in place $M$ obtained by using the measured thickness of the mudstone source rock of each well, and the length of the well. From the upper bound on $M$ one obtains through equation 12 an upper bound on $\tau$, shown in the lower left. This bound on $\tau$ is not very tight. A peak occurs at around 30 yr, but a long tail stretches into the hundreds of years. We think it is impossible that wells will last this long before beginning to interfere, but they are simply too young to provide evidence that interference will occur any sooner.

Finally, in Figure 14 we display information about each of the 2133 wells for which interference can be detected with some certainty. However, rather than directly reporting the original gas in place $M$ and the interference time $\tau$, as done in Patzek et al. (2013), we rescale the presentation of the interference time $\tau$ using equation 7 to calculate the effective spacing $d$ between fracture planes. To do this we need the value of $\alpha_i$, and for the purposes of this computation we assume $\alpha_i = 3 \times 10^{-8}$ m$^2$/s, which corresponds to a permeability $k$ of one nanodarcy and porosity of around 6%. The characteristic spacing comes out to be around 2 m (6.6 ft); this is 10 times smaller than the separation between fracture stages, but is on the order of spacing between natural fractures. Equivalently, one could view the wells as containing fracture stages spaced by 20 m (65.6 ft), but with effective permeability 100 times larger, on the order of 100 nanodarcys.

**DISCUSSION**

1. A simple universal model of pressure diffusion between absorbing boundaries provides surprisingly good agreement with all wells that can reasonably be analyzed in the Barnett Shale and in other shales, for example, Male et al. (2014). The simplicity of the model is particularly surprising because the hydrofracturing process should be expected to produce a complex fracture network with structure on many scales, as described for example by Marrett (1996). Nevertheless, the existing data are fit rather well by treating wells as being in contact with a medium with uniform transport properties.

2. Although we have not yet published the results of most of our calculations, we have checked the model on thousands of wells in the Mississippian Fayetteville located in the Arkoma basin of Arkansas, the Jurassic Haynesville located in the Texas–Louisiana Salt Basin, and the Middle Devonian Marcellus located in the Appalachian basin.
Basin. In all cases we were able to arrive at universal scalings of all wells in given play; for the Haynesville shale, see Male et al. (2014).

3. Inserting characteristic values into equations 71 and 72, one deduces rock permeability $k$ of 50 nanodarcys for $\tau$ of 50 yr and 500 nanodarcys for $\tau$ of 5 yr. These values of permeability are 20–200 times larger than the characteristic value of one nanodarcy found for shale core samples in laboratory experiments. This enhanced permeability must result from the hydrofracturing process, which makes it all the more unexpected that a simple model accounts well for gas transport.

The degree to which permeability on the length-scale of wells increases over permeability of centimeter-scale laboratory samples is identical to the general pattern of permeability increase found in large-scale versus small-scale studies of geological formations (Clauser, 1992). However, what we are describing is a man-made rather than purely natural phenomenon; otherwise, the tens of billions of dollars spent to inject water and proppant into horizontal wells in shale would not be needed.

4. We provide upper and lower bounds on time to interference and original gas in place for all the wells. The median lower bound on time to interference is 5 yr and the median upper bound is 100 yr. The bounds on gas in place are somewhat tighter; the lower bound for mean well is 1 Bscf, and the upper bound is 7 Bscf.

5. Given the available data, one cannot provide better bounds on gas production, even in the Barnett Shale with the longest history of production. Pessimists (Hughes, 2013) see only the lower bounds, whereas optimists (Potential Gas Committee, 2013) look beyond the upper bounds. A rigorous economic analysis of the Barnett play, based on the model presented here, has been published elsewhere (Browning et al., 2013a, b, c; Gülen et al., 2013; Ikonnikova et al., 2013).

**CONCLUSIONS**

We have shown that for mostly single-phase gas flow in the Barnett Shale, the classical models of transient flow in hydrofractured wells (see, e.g., Patzek, 1992), apply, at least during early production. Given the mystique surrounding gas (and oil) production from mudrock systems, this finding was somewhat of a surprise. And then there were more surprises. The main conclusions of this paper are as follows.

1. We have formulated a complete 1-D model of flow of natural gas that can sorb on the rock. The result is a dimensionless, nonlinear diffusion equation of gas pseudopressure. We solve this equation numerically using a fast implicit solver. Our model can be applied to vertical wells, but here we are mostly interested in horizontal wells.

2. Cumulative gas production follows a nearly universal function scaled by two parameters, interference time $\tau$ and mass of gas in place $M$. For over 8000 wells in the Barnett Shale, selected simply because they were horizontal wells that had not been recompleted and had over 18 months of production, agreement with the scaling function is excellent.

3. We have shown that early (but not too early) in the flow, this nonlinear pseudopressure equation leads to a square-root phase, during which gas production rate must decline as $1/\sqrt{t}$ and cumulative production must increase as $\sqrt{t}$.

4. Within the first 1–3 months from starting a field well, gas production is substantially hampered relative to an equivalent ideal horizontal well. The lower gas production in the field is most likely caused by fracturing water left in the hydrofractures and blocking gas flow. We make slight adjustments (time-shifts, for example) of field production to account for this phenomenon.

5. When our dimensionless time, measured in units of $1/2$ of the distance between hydrofractures squared divided by the initial hydraulic diffusivity of gas, approaches 0.64, gas flow starts to deviate measurably from the square-root phase, and enters the interference phase. As the dimensionless time passes 1, gas production rate begins exponential decline. These production regimes are universal; all gas wells in the Barnett Shale that were not recompleted in fact appear to follow them.

6. We have tested this model on over 8807 horizontal wells in the Barnett Shale with at least 18 months of production. We found 2133 wells that had entered the interference phase, and where production had started to decline exponentially.
in less than 10 yr. The remaining 6172 horizontal wells were in the square root phase.

7. In horizontal wells in the Barnett Shale gas flows into transverse vertical hydrofractures that appear as parallel planes, but are likely complex systems of multiscale, nested flow paths that influence a non-negligible volume of reservoir rock around the fracture planes. This complexity manifests itself as an apparent formation permeability that is 1–2 orders of magnitude higher than lab-measured shale permeabilities. Based on our model, we can place both upper and lower bounds on the effective permeability. Lower bounds on \( \tau \) imply upper bounds on the permeability \( k \), whereas upper bounds on \( \tau \) imply lower bounds on the permeability. Using the typical values of Table 1, we find that \( \tau \) of 5 yr corresponds to \( k \) of 500 nanodarcys, 100 yr corresponds to 25 nanodarcys, and 500 yr corresponds to 5 nanodarcys. Thus inspection of Figure 13 shows that permeabilities are typically bounded between 25 and 500 nanodarcys.

8. In vertical wells in the Barnett Shale, each stacked vertical hydrofracture drains its own reservoir interval and acts almost independently of other hydrofractures for 10 to 12 yr in production.

9. Our universal dimensionless solution is translated into production of a composite median horizontal well in the core area of the Barnett. The best match of field data is achieved if this well is assumed to be 1220 m (4003 ft) long and nominally it has 10 hydrofractures spaced every 125 m (410 ft). It is established that the rock permeability is close to 400 nanodarcys, and that each fracture has an effective one-sided area of 7000 square meters (75,350 ft²). Thus, a typical total two-sided area of hydrofractures in a Barnett well is 0.14 km² (0.05 mi²).

10. It appears that gas sorption is negligible in the core area of the Barnett Shale (Browning et al., 2013b, c). However, our still unpublished modeling shows that gas desorption is important in other U.S. shale plays, for example, in the Fayetteville and Woodford Shales, as discussed in Appendix 2.

11. The core-measured values of shale permeability at the length scale of millimeters cannot describe gas flow at the scale of tens or hundreds of meters. At the latter scale, natural and induced fracture systems dominate the flow. Because basic reservoir characterization of the Barnett Shale depends mostly on core analysis (Montgomery et al., 2005), this flow would be missed by the laboratory experiments and the permeability would routinely be underestimated.

12. Pressure interference explains the accelerated decline of gas production at later times. Therefore, with the model we have studied, no need arises for an arbitrary switch from one type of empirical extrapolation of production to another.

**APPENDIX 1: ONE-DIMENSIONAL TRANSIENT FLOW OF REAL GAS**

**Model Formulation**

To arrive at the pressure diffusion equation, we assume that only gas is flowing, and we perform the mass balance for the gas:

\[
-\frac{\partial (\rho_g u_g)}{\partial x} = \frac{\partial}{\partial t}[(\rho_g S_g \rho_a + (1 - \phi)\rho_a)] \frac{kg \text{ gas}}{m^3 \text{ s}}
\]  

wherein which \( u_g \) is the Darcy (superficial) velocity of gas, \( S_g = 1 - S_w \) is gas saturation (\( S_w \) being the connate water saturation), \( \rho_g \) is the free gas density, \( \rho_a \) is the adsorbed gas density (kg/gas/m³ solid), and \( \phi \) is the rock porosity.

By applying Darcy’s law to the 1-D horizontal flow of gas, we can substitute

\[
u_g = -\frac{k}{\rho_g} \frac{\partial p}{\partial x}
\]  

in which \( p \) is the gas pressure, and obtain the following nonlinear partial differential equation:

\[
\frac{\partial}{\partial x} \left( \frac{k \rho_g}{\mu_g} \frac{\partial p}{\partial x} \right) \approx \phi S_g \frac{\partial \rho_g}{\partial p} \frac{\partial p}{\partial t} + (1 - \phi) \frac{\partial \rho_a}{\partial p} \frac{\partial \rho_g}{\partial t}
\]
The absolute permeability of the rock is $k$, and $\mu_g$ is the viscosity of gas, and we have neglected the pore space compressibility:

$$c_f = \frac{1}{\phi} \left( \frac{\partial \phi}{\partial p} \right)_{T=\text{const}} \approx 0 \quad (19)$$

The gas density is related to its pressure and temperature through an equation of state for real gases:

$$\rho_g = \frac{M_p}{Z_g RT} \quad (20)$$

in which $Z_g(p,T)$ is the compressibility factor of gas, $M_g$ is the pseudo molecular mass of gas, $R = 8314.462$ J/kmol⋅K is the universal gas constant, and $T$ is a constant temperature of the reservoir.

The isothermal compressibility of gas is defined as:

$$c_g = \frac{1}{\rho_g} \left( \frac{\partial \rho_g}{\partial p} \right)_{T=\text{const}} = \frac{1}{p} \frac{1}{Z_g} \frac{\partial Z_g}{\partial p} \quad (21)$$

We define $K_a(p,T)$ as the differential equilibrium partitioning coefficient of gas at constant temperature; see, for example, Cui et al. (2009):

$$K_a = \left( \frac{\partial \rho_g}{\partial \phi} \right)_{T=\text{const}} \quad (22)$$

By inserting equations 21 and 22 into equation 18, the general nonlinear equation of transient, 1-D horizontal flow of gas is obtained:

$$\frac{\partial}{\partial x} \left( k \frac{\partial \rho_g}{\partial p} \right) \frac{1}{\mu_g} \frac{\partial p}{\partial x} = \frac{1}{2} \frac{\phi S_g}{\rho_g} \frac{\partial \rho_g}{\partial t}$$

$$K_a \left[ (1 - \phi) K_a \right] \left[ \rho_g \frac{\partial \rho_g}{\partial t} \right] = \frac{\partial}{\partial x} \left( k \frac{\partial \rho_g}{\partial x} \right) \frac{1}{\mu_g} \frac{\partial p}{\partial x} \quad (23)$$

Note that only if

$$q \rho_g S_g$$

is not much greater than $(1 - \phi) K_a$ (24) desorption matters. For the Barnett Shale at or above 100 psi (0.7 MPa), this gas adsorption term is an order of magnitude smaller than $\phi S_g$, and gas adsorption can be neglected, see Appendix 2.

Notice that the hydraulic diffusivity of a gas $\alpha$ strongly depends on the gas pressure, because the gas viscosity $\mu_g(p)$ can vary three-fold and the gas compressibility $c_g \approx 1/p$.

**Table 2. Langmuir Adsorption Isotherm Parameters**

<table>
<thead>
<tr>
<th>Sample*</th>
<th>19HVab</th>
<th>22Vab</th>
<th>26Vab</th>
<th>31Vcd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (ft)</td>
<td>8560.0</td>
<td>8592.0</td>
<td>8628.2</td>
<td>8641.8</td>
</tr>
<tr>
<td>TOC (%)</td>
<td>3.4</td>
<td>3.6</td>
<td>4.0</td>
<td>5.7</td>
</tr>
<tr>
<td>$p_L(T_0)$ (psia)</td>
<td>1281</td>
<td>702</td>
<td>1596</td>
<td>335</td>
</tr>
<tr>
<td>$v_L(T_0)$ (scf/ton)</td>
<td>47.4</td>
<td>55.0</td>
<td>39.2</td>
<td>45.4</td>
</tr>
<tr>
<td>$p_0(T)$ (psia)</td>
<td>2795</td>
<td>1532</td>
<td>3483</td>
<td>731</td>
</tr>
<tr>
<td>$v_0(T)$ (scf/ton)</td>
<td>31.1</td>
<td>36.1</td>
<td>25.7</td>
<td>29.8</td>
</tr>
</tbody>
</table>

*Per personal communication from M. Zoback (2013), all measurements were performed at room temperature, assumed here to be $T_0 = 20^\circ\text{C}$ (68°F). The reservoir temperature is $T = 87^\circ\text{C}$ (189°F).
In addition, both the rock porosity \( \phi \) and the permeability \( k \) are functions of effective stress. In particular, the rock matrix permeability may decrease by an order of magnitude when the pore pressure is depleted (Vermylen, 2011). An example of typical variability of \( \alpha \) is given in Appendix 3.

This nonlinear differential equation 23 is now be simplified by introducing the Kirchhoff integral transform of gas pressure after Al-Hussainy et al. (1966), which in the present context is also called the real gas pseudopressure:

\[
m(p) = 2 \int_{p_0}^{p} \frac{pd\tilde{p}}{\tilde{p}Z_g}\]

Here, \( p_0 \) is a reference pressure that will be set to a constant pressure in the hydrofractures. Appendix 3 contains the relevant physical properties of natural gas and the calculation of the pseudopressure.

Next, to find the gas flowing into a fracture plane we replace the pressure with the real gas pseudopressure in equation 26:

\[
\frac{\dot{m}}{M_g} = \frac{2HLk}{2RT} \left. \frac{\partial \tilde{m}}{\partial \tilde{x}} \right|_{|0|}
\]

A detailed solution of these equations is reported in Patzek et al. (2013) and its supplementary materials. Our primary finding is that the solutions take on universal (or nearly universal) form once we define dimensionless time, distance, and pseudopressure by

\[
\bar{t} = \frac{t}{\tau}; \quad \bar{x} = \frac{x}{d_{\bar{x}}}; \quad \bar{m} = \frac{1}{2} \left( \frac{c_g p}{\mu_g Z_g} \right) \left( \frac{p}{p_0} \right) \tilde{m}(x, t)
\]

Here, the subscript \( i \) refers to the quantities at the initial reservoir pressure \( p_i \) and temperature \( T \). As shown in Figure 26, the factor \( \left( \frac{c_g p}{\mu_g Z_g} \right) \) is close to unity.

The net result is the following dimensionless boundary-value problem: Consider the 1-D flow of gas into a transverse planar hydrofracture of height \( H \), length \( 2L \), and separated by

\[
0 \leq x \leq 2L
\]

\[
0 \leq \bar{t} \leq 1
\]

\[
0 \leq \bar{x} \leq 1
\]

\[
0 \leq \bar{m} \leq 1
\]

The differential gas adsorption for Langmuir adsorption isotherm at the reservoir temperature of 87°C (189°F). Note that \( (1 - \phi)k_a \ll \phi S_g \) and gas adsorption does not matter for the Barnett Shale.
distance $2d$ from the next hydrofracture planes, as depicted in Figure 1. The scaled transport equation is

$$\frac{\partial \tilde{m}}{\partial \tilde{t}} = \alpha \frac{\partial^2 \tilde{m}}{\partial \tilde{x}^2}; \quad \tilde{m}(\tilde{x}, 0) = \tilde{m}_i(\tilde{x}) \quad \tilde{m}(\tilde{x}, \tilde{t}) = 0 \text{ for } \tilde{x} = 0 \text{ and } \frac{\partial \tilde{m}}{\partial \tilde{x}} = 0 \text{ for } \tilde{x} = 1$$

(29)

**APPENDIX 2: SORPTION OF NATURAL GAS**

Natural gas, which is 80–90% methane, adsorbs only on the surfaces of pores in kerogen filaments in a mudrock, and is in thermodynamic equilibrium with free gas in the pores. Langmuir (1918) developed a sorption isotherm to describe this type of equilibrium at a specific temperature:

$$\bar{v}_a = \frac{\bar{v}_L p}{p + p_L} \text{ standard } \frac{\text{cm}^3}{\text{g of bulk rock}}$$

(30)

Here, $\bar{v}_a$ is the specific sorbed gas volume at stock tank conditions (ST) of 1 atm (0.1 MPa or 14.7 psi) at $T_{ST} = 288.7 \text{ K (60°F or 15.5°C)}$; $p$ is the pressure of free gas; $\bar{v}_L$ is the specific volume of sorbed gas at infinite pressure, or the Langmuir volume, also at standard conditions; $p_L$ is the pressure at which 1/2 of the maximum sorbed gas is still adsorbed, or the Langmuir pressure.

The field units of specific sorbed-gas volume are standard cubic feet per short ton (scf/ton), and the conversion factor is

$$1 \text{ scf/ton of bulk rock} = \frac{1}{32} \text{ standard } \frac{\text{cm}^3}{\text{g of bulk rock}}$$

(31)

**Figure 20.** In the cooler and shallower reservoirs with more adsorbed gas, as is the case in the Fayetteville Shale, gas desorption is more significant. As an example, the Langmuir volumes measured by Vermyleen (2011) for the Barnett Shale samples were increased by 50%. For the GBP 1-21 sample, we assumed $\bar{v}_L(T_0) = 86 \text{ scf/ton}$ and $p_L(T_0) = 718 \text{ psia (5.0 MPa)}$, so that their values at the reservoir temperature would be 66 scf/ton and 800 psia (5.5 MPa), respectively, in agreement with table 1 in Boulis et al. (2013). All other reservoir parameters were from the Fayetteville core measurements by Southwestern Energy, and the core temperatures were estimated from the geothermal gradient.

**Figure 21.** Gas-swollen Woodford cuttings bags: doubled plastic bags inside cloth bags. This is how the volume of gas evolved from the core can be measured. Image source: Breig (2010).
Core analysis is required to generate a Langmuir isotherm. However, generally only one Langmuir isotherm is necessary to adequately describe a gas shale within a field or subbasin. The Langmuir volumes and pressures were measured by Vermylen (2011) on four Barnett core samples; see Table 2. The room temperature isotherms are plotted in Figure 17 and the elevated reservoir temperature ones in Figure 18.

The adsorbed gas density at the reservoir temperature is

\[ \rho_a = \hat{v}_a(p, T)\rho_b(p_{ST}, T_{ST})\rho_b \]  

in which \( \rho_b(p_{ST}, T_{ST}) = 7.4 \times 10^{-4} \, g/cc \) (4.3 \times 10^{-4} oz/in.\(^3\)) is the stock tank gas density and \( \rho_b = 2.5 \, g/cc \) (1.4 oz/in.\(^3\)) is the mudrock bulk density (Vermelen, 2011). Consequently,

\[ K_a = \frac{\partial \rho_a}{\partial \rho_b} = \rho_b(p_{ST}, T_{ST})\rho_b \frac{\partial \hat{v}_a(p, T)}{\partial p} \frac{\partial p}{\partial \rho_b} \]

\[ = \frac{\rho_b(p_{ST}, T_{ST})\rho_b \hat{v}_L(T)p_v(T)}{c_p\rho_b} \frac{\hat{v}_L(T)p_v(T)}{(p_v(T) + p)^2} \]  

From Figure 19 it is clear that for the reservoir pressures encountered during gas production in the Barnett Shale, the term \( (1 - \phi)K_a \) is an order of magnitude smaller than \( \phi \rho_a \), and gas adsorption can be neglected. Note that in coal seams adsorbed methane may be as high as 200–400 scf/ton (Dallegge and Barker, 2013).

Because our theory is universal, it applies to all gas shales, some of which may be strongly sorbing gas. In shallower and cooler reservoirs with more adsorbed methane, gas desorption may be more important, see Figure 20. For example, assuming that the Woodford and Fayetteville shales are similar, one can estimate the in situ gas content in the latter from the drilling cuttings in the former, shown in Figure 21. The shape of Langmuir isotherm in Figure 22 is approximate, as \( p_v \) is unknown, and \( v_L \) is approximated with the total evolved-gas volume. With these assumptions, gas desorption is important in the Woodford shale and—by implication—the Fayetteville shale, as can be seen from Figure 23.

### APPENDIX 3: NATURAL GAS PROPERTIES

The composition of a sample of natural gas representative of the core area in the Barnett Shale (compare Hill et al., 2007) is listed in Table 3. The measured gas viscosities and densities at 190°F (88°C), close to a typical reservoir temperature in the Barnett Shale, are listed in Gonzalez et al. (1970), table III-9. To calculate

<table>
<thead>
<tr>
<th>Component</th>
<th>Mole %</th>
<th>( M ) kg/kmol</th>
<th>( T_c ) K</th>
<th>( P_c ) bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methane</td>
<td>91.5</td>
<td>16.043</td>
<td>190.4</td>
<td>46.0</td>
</tr>
<tr>
<td>Ethane</td>
<td>3.1</td>
<td>30.070</td>
<td>305.4</td>
<td>48.8</td>
</tr>
<tr>
<td>Propane</td>
<td>1.4</td>
<td>44.094</td>
<td>369.8</td>
<td>42.5</td>
</tr>
<tr>
<td>n-Butane</td>
<td>0.5</td>
<td>58.124</td>
<td>425.2</td>
<td>38.0</td>
</tr>
<tr>
<td>iso-Butane</td>
<td>0.7</td>
<td>58.124</td>
<td>408.2</td>
<td>36.5</td>
</tr>
<tr>
<td>CO(_2)</td>
<td>1.7</td>
<td>44.010</td>
<td>304.1</td>
<td>73.8</td>
</tr>
<tr>
<td>N(_2)</td>
<td>0.6</td>
<td>28.013</td>
<td>126.2</td>
<td>34.0</td>
</tr>
<tr>
<td>Pentanes</td>
<td>0.6</td>
<td>2.151</td>
<td>469.7</td>
<td>33.7</td>
</tr>
</tbody>
</table>

*Sample No. 3, page 31 in Gonzalez et al. (1970). The critical properties are from Poling et al. (2001).
\( \mu_g(p, T) \) and \( \rho_g(p, T) \), we use a bilinear lookup table. Therefore, we have also used table III-8, “Viscosity of Natural Gas Sample No. 3 at 100, 130, and 160 deg F.” In addition, we wrote an equation-of-state-based pressure versus temperature (pressure-volume-temperature) package to calculate thermodynamic properties of natural gas mixtures of arbitrary composition. This package and the numerical approach will be discussed in a future paper.

From the measured data, one can calculate the gas compressibility factor directly from equation 20:

\[
Z_g = \frac{pM_g}{\rho_gRT}
\]  

(34)

in which \( M_g = 18.82 \text{ kg/kmol} \) is the pseudo molecular mass of the gas, \( \rho_g \) is the measured gas density, and \( T = 361 \text{ K (88°C or 190°F)} \) is the reservoir temperature. With the measured gas viscosity, \( \mu_g \), and the gas compressibility factor calculated from equation 1, one can calculate the gas pseudopressure:

\[
m(p) = 2 \int_{p'}^{p} \frac{p \, dp}{\mu_g Z_g}
\]  

(35)

Here, \( p' = p_f = 500 \text{ psi} \) is the anticipated pressure in the hydrofractures.

The result of numerical integration using the cumulative trapezoid rule is shown in Figure 24.

The numerically integrated equation 35 is well approximated as follows:

\[
m(p) = 4590(p - 500)^{3/2} \text{ Psi}^2/\text{cp}
\]  

(36)

The inverse algebraic transformation,

\[
p(m) = 500 + 0.0036 \text{ m}^2/\text{psi}
\]  

(37)

is shown in Figure 25.

The experimental gas compressibility factor \( Z(p, T) \) can be approximated with a parabola. The isothermal gas compressibility is then obtained by differentiating equation 20:

\[
c_g(p) = \frac{1}{p} \frac{\partial \rho_g}{\partial p} = \frac{1}{p} \frac{1}{Z_g} \frac{\partial Z_g}{\partial p}
\]  

(38)

The result is plotted in Figure 26. If the rock permeability and porosity are held constant and gas adsorption is neglected, the hydraulic diffusivity of gas,

\[
\alpha(p) = \frac{k}{\phi S_p \mu_g(p)c_g(p)} \text{ m}^2/\text{s}
\]  

(39)

increases with pseudopressure because of the gas viscosity, \( \mu(p) \), and gas compressibility \( c_g(p) \). This variability can be captured by calculating the product \( \mu_g(m)c_p(m) \). The result is shown in Figure 27. The inverse of the product of \( \mu_g(m)c_p(m) \) in s\(^{-1}\),

**Figure 24.** The pseudopressure of gas \( m(p) \), obtained by numerical integration of equation 2 and the algebraic fit of \( m(p) \) in equation 3.

**Figure 25.** The gas pressure versus its pseudopressure and the algebraic fit of \( p(m) \) in equation 37.

**Figure 26.** The gas compressibility in equation 38 versus pressure. The compressibility unit is \( 10^6/\text{psi or } \mu \text{ sips} \).
Figure 27. The product \( (\mu c_p) \) \(^{-1} \) versus pseudopressure. Notice that \( \alpha \) varies with pressure as this product. Therefore, over the expected range of reservoir pressures \( \alpha \) varies almost seven-fold.

\[
m' = m \times 10^{-19} \text{ in Pa/s}
\]

\[
\frac{1}{\mu_s(m)c_p(m)} \times 10^{-11} = a_4 m'^4 + a_3 m'^3 + a_2 m'^2 + a_1 m' + a_0 \mu_s c_p \text{ in s} \tag{40}
\]

where \( a_4 = -0.1110, a_3 = 1.0528, a_2 = -3.4362, a_1 = 7.0012, \) and \( a_0 = 2.6869. \)

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