Experimental and Numerical Studies

of One-Dimensional Imbibition in Berea Sandstone

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ABSTRACT

This paper presents the results of a one-dimensional imbibition experiment in a dry Berea sandstone core, along with Computerized Tomography (CT) visualization of the saturation, and a new numerical model of the imbibition. The main aspects of the paper are: (1) a one-dimensional spontaneous imbibition experiment to measure the cumulative imbibition, permeability, and capillary pressure from the slope of saturation vs. square root of time curve; (II) a visual representation of the distribution of water saturation based on CT scan images; and (III) mathematical modeling and numerical simulations to compare with the experimental results.

The experimental study consisted of measuring the cumulative imbibition in a cylindrical Berea sandstone core from the weight change of the core during water imbibition. From the slope of line of the cumulative volume of water imbibed vs. square root of time, the hydraulic conductivity and capillary pressure are calculated. Computerized Tomography has been used to image the water saturation. Using this technique, attenuation differences as small as 0.1%, with a cross-sectional resolution of 1 mm^2 were achieved. Water imbibition has been imaged along a longitudinal cross-section of cylindrical Berea cores and slices have been generated every 40 seconds.

Finally, a mathematical model has been developed to simulate the imbibition. The model assumes a logarithmic capillary pressure curve and a power expression for the air-water relative permeability, with the exponent $n \geq 1$. For n=1, the partial differential equation governing the flow becomes linear and was solved analytically. For n>1, we solve the nonlinear PDE numerically. In both cases, the cumulative imbibition volume and the saturation profiles are calculated as a function of time and compared with the experiments. The linear case shows a proper square root of time behavior, including the late-transient bending of the cumulative water curve, but not the sharp front moving through the core

and shown by the CT experiments. For, $n \cong 2.5$, excellent agreement with both the weight change and CT experiments was obtained.

INTRODUCTION

In this project, our main aim is to understand the role of capillary forces in the displacement process of air by water from a porous media, and to obtain time scales for imbibition in such cases. The specific goals of the study are, (I) to understand the behavior of water front movement in a porous medium, (II) to check the validity of linear relationship between saturation and square root of time during early stages of imbibition, (III) to obtain water saturation distribution in the one-dimensional imbibition case, and (IV) to verify experimental results using our numerical model. To accomplish our task, we conducted experiments focusing on one-dimensional imbibition of water in an air saturated Berea sandstone core. The experiment was conducted weight gain experiment to obtain the times involved in imbibition of water, specifically to know the time scale for capturing the early dynamics of water imbibition. Results from the plot of water saturation vs. square root of time were used to determine the capillary pressure and permeability of the Berea core, and change in average saturation with time. However results from the above experiment provided no information on the distribution of water and air inside the core.

We used the time scales of imbibition from the weight gain experiment to time the scanning of imbibition process using a computerized axial tomography (CT) scanner, so as to capture the early imbibition dynamics. In this study we use high resolution X-ray computerized tomography to obtain longitudinal cross-section images of the water and air in a Berea sandstone core as a function of time. It would have been desirable to obtain both the spatial and temporal distribution of fluids inside the Berea core. However due to CT scanner limitation, in terms of time required to generate a single image, we could obtain images in time at only one location. To obtain fluid saturations in the core as a function of time, scans in a plane passing through the axis of the core were made using a high resolution EMI 5005 (second generation) CT scanner. Images were generated every 40 seconds**.** We used CT technique for flow visualization in this study because it is a very fast, accurate technique with few restrictions on experimental conditions. This method offers fine spatial resolution (attenuation differences as small as 0.1% with a cross sectional resolution of less than 1 mm^3 can be realized). Finally results obtained from the 1-D imbibition experiment and flow visualization studies using CT methodology were compared to that obtained from our numerical and mathematical model.

ONE-DIMENSIONAL IMBIBITION EXPERIMENT

Our main focus of conducting the one-dimensional imbibition experiment was to obtain the change of average saturation with time, time rate of imbibition, and to calculate capillary pressure and relative permeability from the slope of average saturation vs. square root of time. For this study, we used Berea sandstone with a porosity of 22% and a permeability of 300 md. The Berea core, 5.46 cm in diameter and 6.7 cm in length, was fired at 750^0C for 24 hours to remove the effects of clay swelling and migration during the imbibition process. The core was then epoxied on the sides. Excessive care was taken to perfectly seal the sides to obtain one-dimensional imbibition. The fluid used for the study was normal tap water at room temperature. **Fig. 1** is a schematic of the experimental setup. The Berea core is suspended by means of a steel wire from a weighing balance in an acrylic container which can be tightly fitted to the bottom of the weighing balance assembly by means of an o-ring. The acrylic container is connected to a water tank through a rubber tube. The rubber tube opening into the acrylic container has an air filter attached to it to trap air bubbles entrapped in water entering the container. The weighing balance is connected to a data acquisition software configured to obtain weight readings every second.

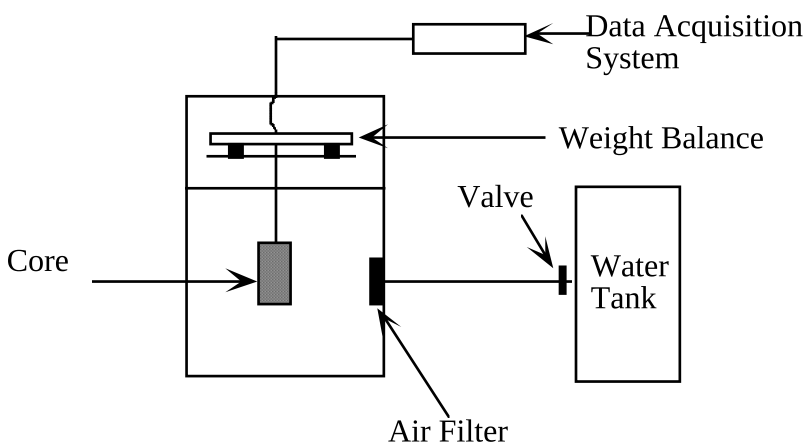

Fig. 1 - Schematic of Experimental Setup

At the onset of the experiment, the data acquisition system was turned on and the readings obtained provided the dry weight of the core. Later the acrylic container with Berea core suspended in it, was filled with water by opening a valve connected to the water tank. The water level in the container was regulated throughout the experiment so as to just touch the bottom of the core. This was done to minimize the influence of the buoyant force on the weight change of the Berea core, arising from submergence of core in water. Readings acquired represented the change in weight of Berea with time. Data was acquired for 120 minutes after water started imbibing into the core. At this point, we shut off the data acquisition system, as no appreciable change in weight of Berea core was noticed. A test of reproducibility was performed on the same core.

FLOW VISUALIZATION USING CT METHODOLOGY

Data obtained from 1-D imbibition provided time rate of change of average saturation inside the core. However we could not obtain any information on the saturation distribution of water and air in the core. CT methodology was employed to map fluid distribution in a longitudinal section of the core

every 40 seconds. The average saturation of water obtained from CT was later compared to that obtained from 1-D imbibition test. Reference of earlier works illustrating application of CT methodology in flow visualization studies is obtained from petroleum literature [3].

The apparatus shown in **Fig. 2** was used to image the Berea core using an EMI 5005 (second generation) CT

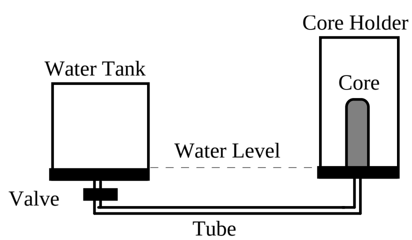

Fig. 2 - Schematic of CT Scan Experiment

scanner at Stanford University. The cores were scanned at an energy level of 140 keV and a field size of 13 cm. A small field of scan was used to obtain better spatial resolution as the number of pixels available remain constant. Slice thickness was made as small as possible, i.e., 3 mm in order to minimize errors and maximize resolution. A scan angle of 398^o was used as it produces the highest resolution due to an overscan of 38^o. Our experimental cell is constructed from acrylic material, which is X-ray transparent so that it does not yield excessively high attenuation values.

A porous Berea sandstone core, 6.7 cm in length and 5.46 cm in diameter was concentrically positioned inside the core holder, placed vertically inside the scanner gantry. The core holder 6.4 cm in diameter and 21 cm long, has two end caps for fluids to flow in and out of the core holder. The inlet endcap at the bottom is connected to a fluid tank through a rubber tube. A control valve attached to the tank controls the flow of fluid from tank to coreholder. The outlet endcap is connected to a measuring vessel. At the commencement of the experiment, the first images to be scanned were that of an air saturated dry core at an initial pressure of 1 atm, to obtain dry core CT values (CT_{dry}). Later, water was allowed to imbibe in the dry core by opening the valve. Excessive care was taken to maintain water level in core holder just touching the rock base. X-ray scanning was done at the same longitudinal section of the core to obtain CT values (CT_{exp}) at intervals of 40 seconds for 520 seconds and later. A final scan was conducted at 24 hours after start of the experiment. The weight of the core was measured at the beginning and end of the experiment for mass-balance calculations. Eq. (1), applied to each pixel of the slice is used to calculate water saturation in any slice of the core.

$$S_{water} = \frac{CT_{water} - CT_{\exp}}{CT_{water} - CT_{dry}} \qquad \textbf{\textit{(1)}}$$

CT_{water} is the CT value for a fully water saturated core. In this study, slices obtained from X-ray scanning after 12 hours of water imbibition were used as the fully water saturated core CT values, CT_{water}. From mass-balance calculations, the final water saturation in the core was found to be 85%. Thus for obtaining accurate saturation values, water saturation values obtained from Eq. (1) were multiplied by a factor of 0.85.

ONE-DIMENSIONAL MATHEMATICAL MODEL

In this section a one-dimensional imbibition model will be presented. We use this model to simulate both the weight gain and the CT experiments previously discussed. Earlier studies in this area have also focused on developing a numerical model for case of 1-D spontaneous imbibition (Zwahlen, 1995), (Firoozabadi, 1994).

Starting with the one-dimensional continuity equation and Darcy's law for air and water, and assuming the air has infinite mobility, we derive the following equation for the water saturation.

$$\phi\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial z}\left(\frac{kk_{rw}}{\mu_w}\left(\frac{dp_c}{dS_w}\frac{\partial S_w}{\partial z} - \Delta\rho g\right)\right) = 0 \qquad \textbf{\textit{(2)}}$$

We now introduce the normalized saturation

$$S = \frac{S_{w-S_{wc}}}{1 - S_{wc} - S_{gr}} \qquad \textbf{\textit{(3)}}$$

where S_{wc} is the connate water saturation and S_{gr} is the residual gas saturation. The connate water saturation refers to the irreducible and immobile water in the porous medium, thus S ranges from 0 to 1 while S_w ranges from S_{wc} to S_{gr}.

We make two major assumptions to simplify the saturation equation. We assume the relative permeability function can be described by

$$k_{rw} = k_{rw}^{0} S^{n}, \qquad n \geq 1 \tag{4}$$

This means that the water relative permeability is proportional to the normalized saturation raised to some power *n*, where *n* is greater than or equal 1. We also assume the capillary pressure function can be described by a simplified one-parameter expression

$$p_c = p_{c0} \ln S \tag{5}$$

where p_{c0} is an experimental fit parameter. Combining the previous equations gives a non-linear diffusion-advection equation for the saturation,

$$\frac{\partial S}{\partial t} - D\left[S^{n-1}\left(\frac{\partial^2 S}{\partial z^2}\right) + (n-1)S^{n-2}\left(\frac{\partial S}{\partial z}\right)^2 \right] - VnS^{n-1}\frac{\partial S}{\partial z} = 0 \tag{6}$$

where

$$D = \frac{kk_{rw}^{0} p_{c0}}{\mu_w \phi \left(1 - S_{wc} - S_{gr}\right)} \tag{7}$$

and

$$V = \frac{kk_{rw}^{0} \Delta\rho g}{\mu_w \phi \left(1 - S_{wc} - S_{gr}\right)} \tag{8}$$

Here *D* describes the capillary forces and *V* describes the gravity forces. We can also write the Darcy velocity in terms of *S* as

$$u_{wz} = -\frac{kk_{rw}^{0} p_{c0}}{\mu_w} S^{n-1}\left(\frac{\partial S}{\partial z} + \frac{\Delta\rho g}{p_{c0}} S\right) \tag{9}$$

We note that for *n*=1 the differential equation and the velocity equation become linear. The analytical solution is given elsewhere (Zwahlen, 1995). In this paper we will show results of both the linear and non-linear cases.

FINITE-DIFFERENCE SOLUTION

In this section the non-linear differential equation of the previous section will be discretized and a simple, explicit finite difference scheme will be presented. Then numerical results for Berea sandstone will be given and compared with the experimental results.

Let *N* be the number of grid blocks. Then define the number of node points, *NP*1 (*N* plus 1) as

$$NP1 = N + 1 \tag{10}$$

We let j=1 be the bottom row and j=NP1 be the top row. Also, the spatial step size is

$$\Delta z = \frac{L_z}{N} \tag{11}$$

Using forward differencing in time and central differencing in space the differential equation becomes

$$\begin{aligned} S_j^{m+1} &= S_j^m + D\left(\frac{S_{j+1}^m - 2S_j^m + S_{j-1}^m}{(\Delta z)^2}\right)\Delta t \\ &+ D(n-1)\left(S_j^m\right)^{(n-2)}\left(\frac{S_{j+1}^m + S_{j-1}^m}{2\Delta z}\right)^2 \Delta t \\ &+ Vn\left(S_j^m\right)^{(n-1)}\left(\frac{S_{j+1}^m + S_{j-1}^m}{2\Delta z}\right)\Delta t; \\ j &= 2, \ldots, NP1-1 \end{aligned} \tag{12}$$

The superscript m refers to the time step.
At the top boundary the saturation is calculated from the no-flow condition which is discretized as

$$S_{NP1}^{m+1} = \frac{4S_{NP1-1}^m - S_{NP1-2}^m}{\left(3 + \frac{2\Delta z \rho g}{p_{c0}}\right)} \tag{13}$$

The bottom boundary is simply

$$S_1^{m+1} = S_1^m = 1 \tag{14}$$

The solution technique is to choose a time-step size Δt and the number of grid blocks N. It is important to choose Δt small enough for the scheme to be stable. First we calculate the interior points at time step m+1 from the known values at time-step m using the discretized differential equation. Then the boundary points for m+1 are calculated. Then the process can be repeated for the next time-step.

RESULTS

Data obtained from the imbibition experiment was converted into the weight of water imbibed into the Berea core in order to measure the rate change of water saturation in core with time. Weight gain vs. $t^{0.5}$ plot for the Berea sandstone is shown as the bold curve in **Fig. 3**. A fairly straight line is obtained at early stages of imbibition. The straight line portion of the curve is shown by an arrow for reference. The rapid rate of imbibition at early times is due to high permeability and wettability (strongly water wet) of Berea sandstone. At the very start of imbibition, the apparent weight of the core is affected both by buoyancy forces and interfacial forces acting between the core and water. Whereas a buoyancy force contributes to a decrease in the apparent core weight, interfacial forces between the core and the water tend to increase its apparent weight. Through trial and error, a point was finally chosen on the plot which represents the beginning of straight line portion of the plot. In previous works by (Handy, 1960) and (Babadagali, 1992), a similar straight line portion was obtained. However no mention of the spontaneous imbibition phenomenon occurring at very early stages and the bending over of the curve at later stages was made. In those studies, a straight line was statistically fitted to data points on the S vs. $t^{0.5}$ plot and it did not pass through the origin. In this work, we have presented the entire dynamics of spontaneous imbibition of water in an air saturated Berea. The straight line portion represents maximum rate of imbibition, and the deviation from the plot at around 625 seconds points is due to the slowing down of imbibition rate. Later at around 1600 seconds, the plot completely bends over and the rate of imbibition is drastically reduced. At this stage a very slow change in water saturation with time is observed in the core. From mass balance calculations, the final saturation of water in core is 85 %, indicating a residual air saturation of 15 %.

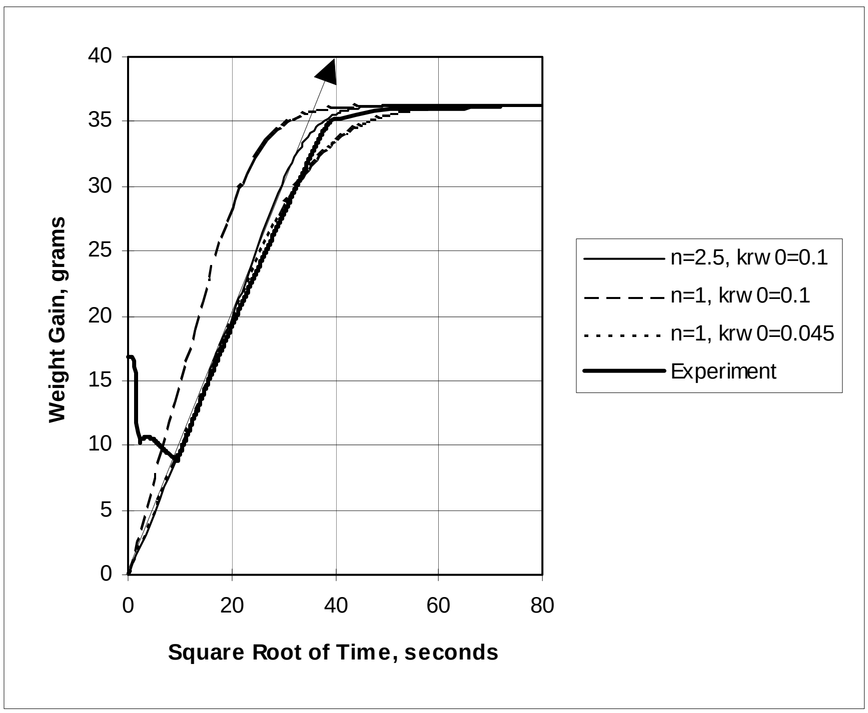

Fig 3 - Weight Gain vs. Square Root of Time

Results from the 1-D imbibition experiment were used to time the imaging of water saturation distribution in the Berea core. In order to visualize the fluids saturation distribution in the core at different times, CT imaging was conducted. **Fig. 4** shows the CT images of water saturation distribution at different time intervals. In Fig. 4, the dark portion represent maximum water saturation with various shades of gray showing intermediate saturations. White represent the dry air saturated Berea core. Images 4a, 4b, 4c, and 4d were imaged continuosly and represents water saturation at 80, 120, 160, and 200 seconds respectively. Images 4e and 4f are the saturation distribution at times 320 and 360 seconds respectively, and 4g and 4h are the images after 1800 and 5400 seconds respectively after the start of imbibition. In all the above images, the water front moves steadily and uniformly, more in a piston like manner. Images 4a-4f correspond to the straight line portion of Fig. 3. The rapid advance of the water front over time indicates a very high rate of imbibition as was seen earlier in Fig 3. Image 4g corresponds to the bending over of plot in Fig. 3 at 1800 seconds. The imbibition front which moves very fast early on slows down considerably in images 4g and 4h. Also both Figs 3 and 4 show that over 90 % of the water saturation is obtained in just 1600 seconds specifying that spontaneous imbibition of water in Berea is a very fast process. Fig. 4 shows the movement and geometry of water front at different times. In order to observe the movement and geometry of water front in more detail, water saturation profiles at different times along the length of the core are shown in Fig. 5

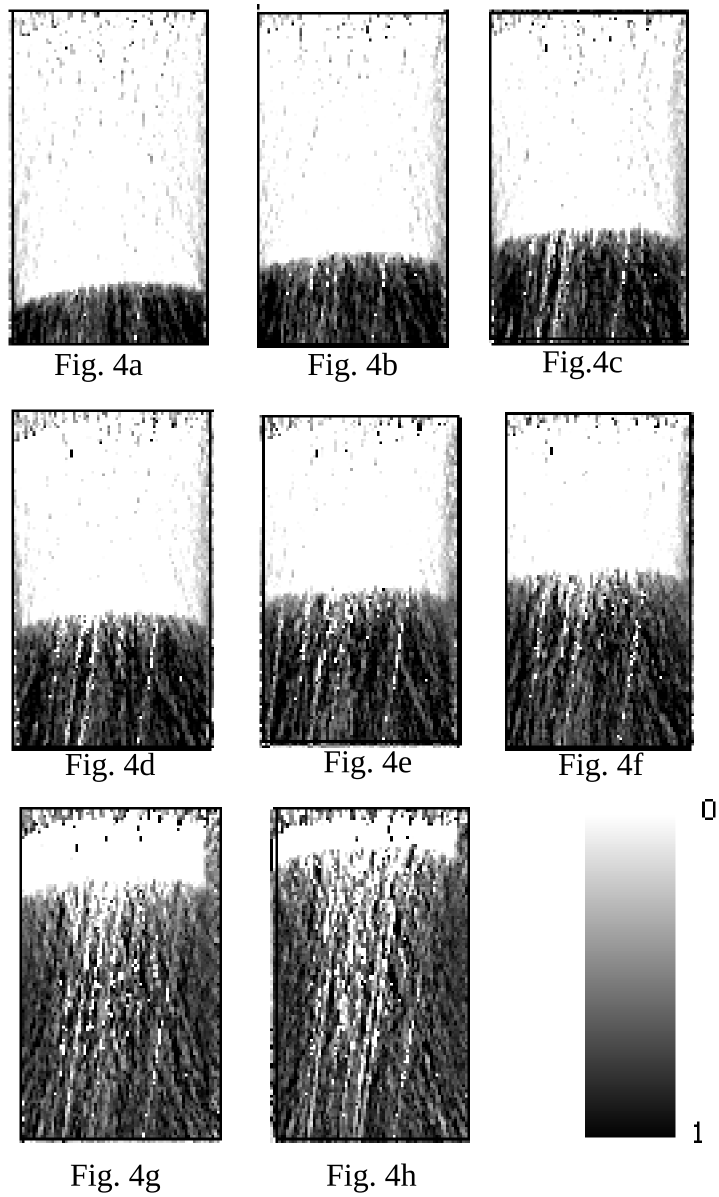

Fig. 4 - CT Images of Water Imbibition in Dry Berea

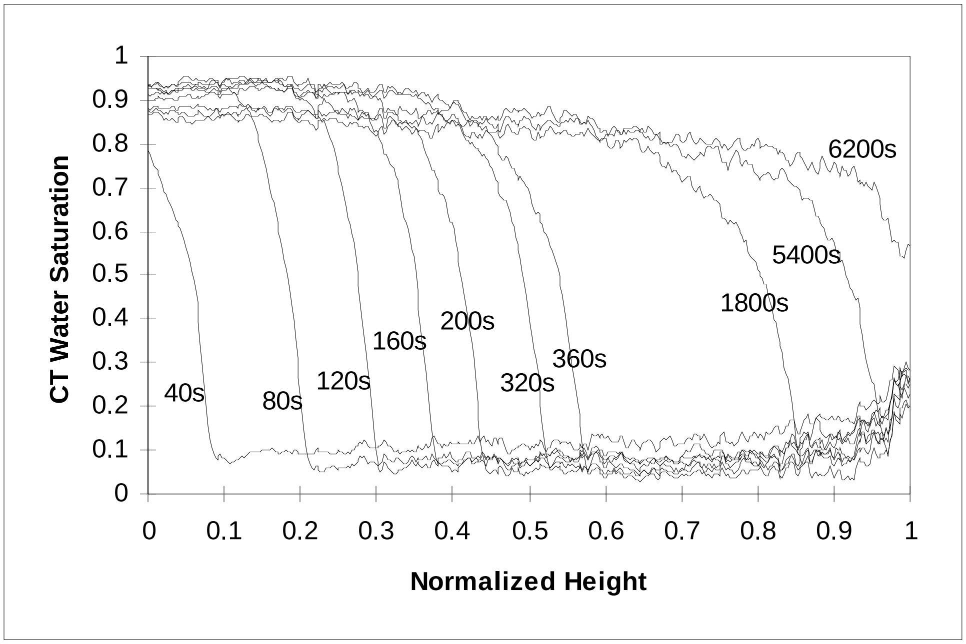

Fig 5 - CT Water Saturation Profiles for Various Times

Once again from Fig. 5 it is clear that a rapidly moving, very sharp water front is obtained during early stages of imbibition. Later at around 1800 seconds, the rapidly moving sharp water front gradually changes into a slow moving, slightly less sharp front.

We now present the results of the numerical simulation. In order to match the weight gain experiment, we chose the following parameter values: k= 300 md, k_{rw}^{0} =0.1, p_{c0} = 7 psi, μ = 1 cp, ϕ = 0.22, n = 2.5, $\Delta\rho$ = 1 g/cm^3, S_{wc} = 0, S_{gr} = 0.15, and g= 980 cm/s^2. For the numerical simulation we chose N = 100 and Δt = 0.01 s. We show the weight gain of the core vs. the square root of time as the solid curve in Fig. 3. These parameters show the proper linear behavior for nearly the entire imbibition. Then around 1600 seconds, the curve flattens out and the imbibition ends.The ending predicted by the numerical model for n = 2.5 is nearly as abrupt as the experiment.

Also shown in Fig. 3 are two dashed curves for n = 1, which are simulations of the linear diffusion/advection equation.. The curve for k_{rw}^{0} =0.1 shows the water imbibes more quickly than the experiment. For k_{rw}^{0} =0.0.045, we can match the early experimental behavior. However, the end of imbibition is not abrupt enough.

In Fig. 6, we show the normalized saturation profiles vs. the normalized height in the core for various time. This is for the non-linear equation with n = 2.5. We see a definite front moving through the core, although the front is not as sharp as shown in Fig. 5, which is very plug like. Notice the rapid movement of the front at early times as it progresses 40% into the core in the first 120 seconds. The front reaches the top of the core in 560 seconds. After 1800 seconds, the imbibition is nearly complete. We also show for comparison a simulation of the linear case with n=1 and k_{rw}^{0} =0.0.045. in Fig. 7. This shows completely different behavior than the non-linear case and the CT experiment. The front moves through the core too rapidly, reaching the top in 120 seconds. Also, the profiles do not show a sharp front, but are convex upward.

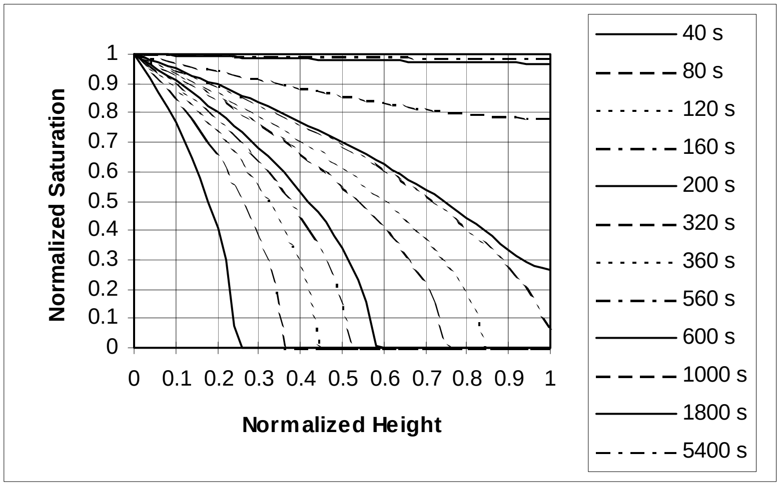

Fig 6 - Normalized Saturation Profiles from Numerical Model (*n*=2.5)

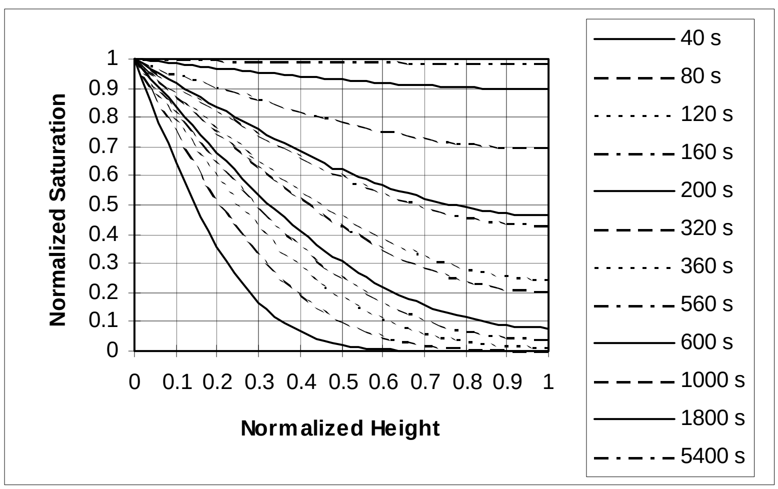

Fig 7 - Normalized Saturation Profiles from Numerical Model (*n*=1)

Conclusions

In this paper we have attempted to present a complete picture of spontaneous imbibition process in a Berea sandstone core. Spontaneous imbibition in the small core studied here is a very rapid process with most of the process occuring in 1600 seconds. CT imaging was done to observe the movement of water front in dry Berea and to determine the water saturation distribution. Images obtained show a homogeneous, very sharp water front at early stages of imbibition, changing gradually into a less sharp front at later times. Excellent matching was found between the non-linear numerical model and the weight gain experiment. The model also showed the same types of profiles as the CT experiment. The liinear model can approximate the weight experiment, but it predicts profiles that are not sharp and water breakthrough at the top of the core that is too fast.

NOMENCLATURE

CT = CT number, Hounsfield
$\Delta\rho$ = water density, 1 gm/ cm^3
Δt = time step, seconds
g = acceleration of gravity, 980 cm/sec^2
k = absolute permeability, 300 md
k^0_{rw} = relative permeability coefficient
L_z = height of core, cm
μ_w = water viscosity, 0.01 gm/cm/sec
n = relative permeability exponent
NP1 = number of grid points in each zone, 101
p_{c0} = capillary pressure coefficient
ϕ = porosity
S = normalized saturation
S_{gr} = residual gas saturation, 0.15
S_{wc} = connate water saturation, 0
S_w = water saturation
S_{water} = water saturation computed from CT values

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